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BUBBLE GROWTH IN THE PRESENCE OF A MAGNETIC FIELD

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Abstract—The problem of the growth of a bubble in the presence of a magnetic field is examined under the assumption that the process is heat transfer controlled. The work reveals the existence of a new nondimensional number which physically represents the ratio of the ponderomotive forces over pressure forces computed on the basis of length-scale and time related to the coefficient of thermal diffusion. It is found that for a spherical magnetic field, growth of the bubble remains parabolic in time but the rate of growth is slower. Heat-transfer estimates are also made in the fashion of Forster and Zuber indicating that heat transfer in nucleate boiling is reduced in the presence of magnetic fields. In a numerical example, this reduction is more substantial for potassium than it is for mercury.

NOMENCLATURE

B, magnetic field intensity;
c_p, coefficient of specific heat at constant pressure;
E, quantity defined in equation (17);
F, quantity defined in equation (17);
i_{fg}, latent heat of vaporization;
J, current density;
Ja, Jacob number defined in equation (15);
Ja^*, generalized Jacob number defined in equation (21);
K, nondimensional number defined in equation (18);
k, coefficient of thermal conductivity;
Nu, Nusselt number;
Pr, Prandtl number;
p, pressure;
R, bubble radius;
R^*, nondimensional bubble radius defined in equation (16);
Re, Reynolds number;
r, radial coordinate;
T, temperature;
u, liquid phase velocity;
V, vapor–liquid interface velocity.

Greek symbols

α, coefficient of thermal diffusivity;
γ, constant defined in equation (12);
Δp, pressure difference defined in equation (18);
ΔT, temperature difference defined in equation (14);
ε, density ratio defined in equation (3);
ν, coefficient of kinematic viscosity;
ρ, mass density;
σ, coefficient of surface tension;
σ_e, electrical conductivity;
τ, time;
τ^*, nondimensional time defined in equation (16).

Subscripts

0, indicates quantity computed at zero magnetic field;
sat, indicates quantity computed at saturation point;
∞, indicates quantity computed far away from the bubble.

Dots on top of symbols indicate differentiation with respect to time.

INTRODUCTION

IT IS well known that many aspects of boiling are difficult to understand. Indeed there are good reasons for more research to be conducted on boiling for the ordinary non-magnetic case before one gets into the complications of the presence of a magnetic field. The motivation for such a challenging excursion, however, comes from technical applications that demand some rudimentary analysis in order to design technologically important devices. Two of them can be mentioned. MHD power generation loops using two-phase flow liquid metals and second fusion reactor blanket designs. In this last case it has been proposed that the plasma heated liquid lithium be used to boil potassium, the vapor of which in turn could be utilized in a power producing topping cycle [1].

Some experimental attempts have been made [2–5] to investigate the heat transfer in nucleate boiling in the presence of a magnetic field, but at best they are contradictory and inconclusive. No theoretical attempt seems to have been made to discuss aspects of this question.

In this paper a simplified model of the bubble growth in the presence of a magnetic field will be developed aiming at getting an insight into the physics of the problem rather than attempting an exact description of the phenomenon. With the help of the conclusions reached from this first part of the investigation, an attempt is then made to estimate heat transfer rates in the simple case of nucleate boiling.

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HYDRAULIC ASPECTS

First we must set the geometry of the magnetic field in relation to the bubble. From a practical point of view it would be appropriate to set it constant and uniform at a given angle with the horizon. Clearly such a configuration would lead, during the bubble's growth, to a non-spherical geometry since growth along the magnetic field would be unrestrained. For a first attempt at understanding the physics of the problem, this geometry provides complications that only extensive numerical work can untangle. Instead, the assumption will be made that the magnetic field is constant and spherical, always remaining normal to the radial direction of growth. This guarantees that spherical symmetry is not lost, however later an account will be made of the fact that this configuration provides a stronger interaction.

Let us now see how the magnetic field is accounted for in the equations of conservation. It is well established that during the initial formation of a bubble a critical size will be reached at the moment when the difference in the pressure of the vapor inside and the liquid outside is balanced by the surface forces. In mathematical language,

$$p_v - p_l = \frac{2\sigma}{R_0}. \tag{1}$$

A moment later when the bubble becomes larger than R_0 the pressure difference will no longer be balanced by surface tension forces but all other forces must be accounted for, such as inertia, viscous and, in our case, the ponderomotive force.

The momentum equation has been developed by numerous authors, such as for instance by Scriven in [6] and is presented below with the addition of the ponderomotive force.

$$p_v - p_l - \frac{2\sigma}{R} = \left(R\ddot{R} + \tfrac{3}{2}\dot{R}^2 + 4\nu\frac{\dot{R}}{R}\right)\varepsilon\rho_l - \int_R^{\infty} (\mathbf{J}\times\mathbf{B})_r \,\mathrm{d}r. \tag{2}$$

In the above R is the bubble radius, σ the surface tension, ρ the mass density, ν the kinematic viscosity and the subscripts v and l refer to the vapor and liquid phases. The quantity ε is defined as follows

$$\varepsilon = \frac{\rho_l - \rho_v}{\rho_l}. \tag{3}$$

For the case of some liquids, but in particular for liquid metals, some simplifications to the Rayleigh equation are possible. For our purposes the viscous forces can be neglected. In addition the surface tension forces, apart from their initial role, do not contribute very much in the subsequent growth.

It remains now to examine the ponderomotive force. We shall assume that the magnetic Reynolds number is very small. Because of the assumed geometry of the magnetic field, the induced current will close on itself in the liquid phase and hence in the frame of reference of the laboratory there will be no electric field. In addition, since at all times the velocity is perpendicular to the field one can deduce by using Ohm's Law that the ponderomotive force per unit volume acting in the radial direction is equal to

$$(\mathbf{J}\times\mathbf{B})_r = -(\sigma_e uB)\cdot B = -\sigma_e uB^2. \tag{4}$$

In the above σ_e is the electrical conductivity and u is the velocity of the liquid phase in the radial direction. If V is the velocity of the interface of the vapor and liquid phase, then simple conservation of mass relates u and V as follows

$$\rho_l ur^2 = V(\rho_l - \rho_v)R^2 = (\rho_l - \rho_v)\dot{R}R^2. \tag{5}$$

The ponderomotive force per unit mass from Ohm's Law is then equal to

$$-\frac{\sigma_e B^2 u}{\rho_l} = -\frac{\sigma_e B^2}{\rho_l}\cdot\frac{\varepsilon\dot{R}R^2}{r^2}. \tag{6}$$

We now integrate this force from $r = R$ to $r = \infty$ to obtain

$$-\frac{\sigma_e B^2}{\rho_l}\varepsilon\dot{R}R.$$

With the above approximations the fundamental equation (2) becomes with $\varepsilon \approx 1$

$$p_v - p_l = (R\ddot{R} + \tfrac{3}{2}\dot{R}^2)\rho_l + \sigma_e B^2 R\dot{R}. \tag{7}$$

We now need to discuss the role of the inertia terms. It is well established that for liquid metals, where the thermal conductivity is high, the inertia terms are very important during the period of initial growth. It is later on, as the bubble has grown considerably, that the growth is heat transfer controlled. On the other hand, inclusion of the inertia terms in the equations leads to computational complications and it will be best to discard them at this time. It should be clear, however, that for high magnetic fields, these inertia forces will be very small compared to the other forces. More discussion of this point follows later in the text. From all of the above, the equation of momentum conservation is approximated as follows:

$$p_v - p_l = \sigma_e B^2 \dot{R}R. \tag{8}$$

Equation (8) needs now to be coupled with the principle of energy conservation. In words, this principle states that the energy conducted into the bubble is equal to the change of the enthalpy necessary for the transition from the liquid to the vapor phase. If the latent heat of vaporization is i_{fg} the above statement can be written as follows:

$$k\frac{\Delta T}{\Delta r} \approx i_{fg}\rho_v\dot{R}. \tag{9}$$

As it is customary in similar problems [7], the length Δr is handled and set to be proportional to $(\alpha\tau)^{\frac{1}{2}}$. We have

$$k\frac{\Delta T}{(\alpha\tau)^{\frac{1}{2}}} = i_{fg}\rho_v\dot{R}$$

or:

$$\dot{R} = \frac{k(T_\infty - T_v)}{i_{fg}\rho_v(\alpha\tau)^{\frac{1}{2}}}. \tag{10}$$

In the above, k is the coefficient of thermal conductivity, α the thermal diffusivity, τ stands for time, and T_∞ is the temperature far away from the bubble.

We now invoke the linearized Clausius–Clapeyron expression that relates pressure to temperature differences in terms of the slope γ in an appropriate thermodynamic diagram. In mathematical form this statement is

$$p_v - p_l = \gamma(T_v - T_{sat}) = \frac{i_{fg}\rho_v}{T_{sat}}(T_v - T_{sat}). \tag{11}$$

Substituting equation (11) in equation (8) we have

$$\gamma(T_v - T_{sat}) = \sigma_e B^2 R\dot{R}. \tag{12}$$

Parenthetically, we will assume that γ remains constant during the growth time. See the recent paper by Theofanous and Patel [8] for an interesting discussion of this approximation.

We can now eliminate T_v from equations (10) and (12) and solve for R to find:

$$\dot{R} = \frac{\gamma(\Delta T)}{\sigma_e B^2 R + \dfrac{\gamma i_{fg}\rho_v(\alpha\tau)^{\frac{1}{2}}}{k}} \tag{13}$$

where

$$(\Delta T) = T_\infty - T_{sat}. \tag{14}$$

We now introduce non-dimensional parameters before seeking a solution of the fundamental equation (13). First we invoke the definition of the well-known non-dimensional Jacob number.

$$Ja = \frac{\rho_l c_p(\Delta T)}{\rho_v i_{fg}}. \tag{15}$$

We nondimensionalize length and time as follows [8]

$$R^* = \frac{R}{E^2/F}, \quad \tau^* = \frac{\tau}{(E/F)^2} \tag{16}$$

where

$$F^2 = \tfrac{2}{3}\frac{\gamma\Delta T}{\rho_l}, \quad E = 2\alpha^{\frac{1}{2}}\cdot Ja. \tag{17}$$

We also introduce a new non-dimensional magneto-fluid-mechanic number from the following relation

$$K = \frac{\sigma_e B^2\alpha}{\gamma(\Delta T)} = \frac{\sigma_e B^2\alpha}{\Delta p} = \frac{\sigma_e B^2\alpha T_{sat}}{i_{fg}\rho_v\Delta T}. \tag{18}$$

If we use all of the above, equation (13) in non-dimensional form becomes

$$\dot{R}^* = \frac{1}{4KJa^2R^* + 2(\tau^*)^{\frac{1}{2}}}. \tag{19}$$

Upon integration we find that

$$R^* = Ja^*(\tau^*)^{\frac{1}{2}} \tag{20}$$

where Ja^* is a modified Jacob number given from the relation

$$Ja^* = \frac{-1+\sqrt{(1+8KJa^2)}}{4KJa^2}. \tag{21}$$

Note that when $K = 0$, $Ja^* = 1$.

Also in the limit of very large K we have

$$Ja^* = 1/Ja(2K)^{\frac{1}{2}}. \tag{22}$$

It is interesting to see from equation (20) that even in the presence of the magnetic field we still get the parabolic growth dependence. This is partly due to our assumption of a spherical magnetic field. We should also note the discovery of the new non-dimensional number K defined in equation (18). Physically K represents the ratio of the ponderomotive force to the pressure forces, computed on the basis of scale and time related to the thermal diffusion coefficient. Equation (21) is now plotted in Fig. 1, where one can see the extent to which the ponderomotive force retards the rate of growth of the bubble.

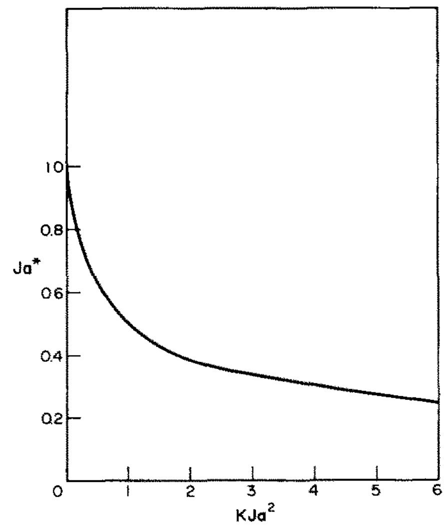

FIG. 1. Variation of generalized Jacob number with KJa^2.

It should be noted that it is the parameter KJa^2 that is a true measure of the magnetic interaction. Using the definitions of K and Ja we find that

$$KJa^2 = B^2(\Delta T)\left[\frac{\sigma_e\alpha\rho_l^2c_p^2}{\gamma\rho_v^2 i_{fg}^2}\right].$$

For a given fluid this parameter is proportional to the square of the magnetic field and the superheat ΔT.

Some recent work to be reported in another paper with Lorry Wagner [10] takes into account the inertia terms. Results show that for zero magnetic field, the inertia controlled solution falls into the heat-transfer controlled case at about $t^* = 10^2$. The equivalent values for the case of $KJa^2 = 10$ and 50 is $t^* = 1$ and 0.1 correspondingly.

HEAT-TRANSFER ASPECTS

It has yet to be established what is the connection between the growth characteristics of a bubble and boiling. Even so, Forster and Zuber [9] have suggested a heat-transfer correlation based on such a connection and although this relation has been disputed it is also true that it has been found capable of correlating some of the available experimental data. The logic of this correlation is simple. Since in heat transfer problems the Nusselt number depends on Reynolds and Prandtl numbers, we need to have, apart from physical properties, a scale for length and a scale for velocity for proper definition of the Reynolds number. Forster and Zuber chose as a scale the instantaneous value of the radius of the bubble and as a velocity the rate of its growth. Since in the heat-transfer controlled region, the radius of the bubble grows with the square root of time, it is obvious that the Reynolds number remains constant with time. This provides the basis for a time independent heat-transfer correlation. In the case of the presence of a magnetic field the parabolic law still holds and hence we can indeed produce a time independent Reynolds number via the parameter K. In terms of the Jacob and Prandtl numbers, Forster and Zuber define the Reynolds number from the above argument as follows:

$$Re = \frac{Ja^2}{2Pr}. \tag{23}$$

In the non-magnetic case they also suggest

$$(Nu)_0 = 0.0015(Re)_0^{0.62}Pr^{0.33}. \tag{24}$$

In our case we need only to modify the appropriate Reynolds number. If we compute the ratio of the Nusselt number with the magnetic field over the Nusselt number without the presence of a magnetic field we obtain the following expression in terms of the generalized Jacob number.

$$\overline{Nu} = \frac{Nu}{(Nu)_0} = \left[\frac{Re}{(Re)_0}\right]^{0.62} = [Ja^*]^{1.24}. \tag{25}$$

In Fig. 2 we can see how this ratio depends on the parameter KJa^2. This provides a universal description of what happens, heat-transfer wise, for all Jacob and K numbers. On the other hand, in Figs. 3 and 4 the dependence of $\overline{Nu}$ is shown for mercury and potassium for different degrees of superheat.

Before enumerating the conclusions as they emerge from the above analysis, it should be stated that the heat-transfer trends as presented here perhaps approximate the case of a horizontal magnetic field. In the case of a vertical magnetic field, it is possible that even though the growth of the bubble will be restricted in the horizontal direction, the eventual rising of the bubbles due to the buoyant forces will be performed along the smooth and orderly vertical trajectories imposed by the magnetic field, thus enhancing the heat transfer to the bulk of the fluid. On the contrary, when the magnetic field is horizontal, buoyancy will be restrained, and more bubbles will remain longer in their nucleation sites, with the possibility of a more direct transition from nucleate to film boiling in this case.

Finally, the following trends evolve from the analysis that lead us to equation (25):

1. The higher the superheat the more effective is the magnetic field in reducing heat transfer. This is evident from the fact that higher superheats are capable of sustaining faster bubble growths. Analytically, the interaction parameter KJa^2 is seen to be proportional to the superheat (ΔT).
2. Because of the nature of physical constants the magnetic field is more effective in reducing heat transfer in potassium than it is in mercury. This is due primarily to the higher thermal diffusivity associated with potassium, hence a higher interaction parameter KJa^2.
3. As it is the case with so many problems of similar nature, the reduction of heat transfer is dramatic at relatively low magnetic fields but at higher magnetic fields the rate of reduction is much less.

The present simplified theory predicts for instance, that in a field of 1 T, heat transfer for a superheat of 10°C is decreased by 5 and 20% for mercury and potassium correspondingly. It should be noted on the other hand, that this corresponds to a spherical magnetic field. If one computes an equivalent magnetic field based on the horizontal projection of the spherical one, one can find that:

$$B_{\text{equivalent}} = \frac{2}{(\pi)^{\frac{1}{2}}} B \approx 1.13B. \tag{26}$$

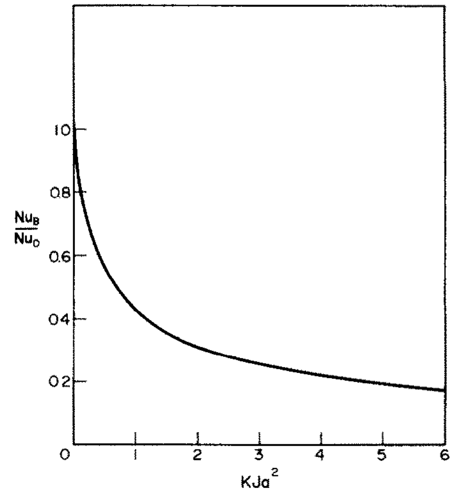

FIG. 2. Nusselt number ratio vs KJa^2.

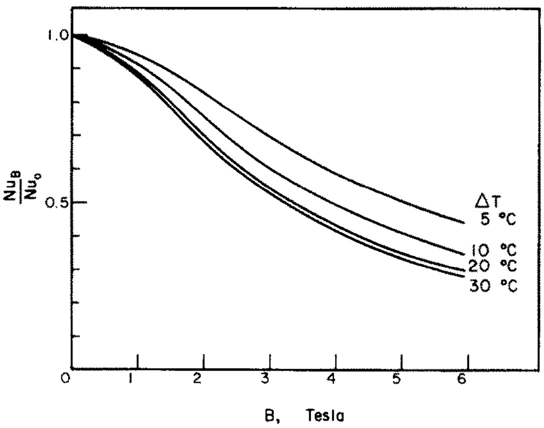

FIG. 3. Nusselt number ratio for mercury.

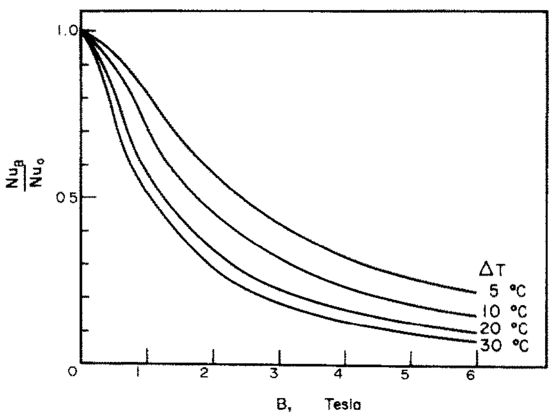

FIG. 4. Nusselt number ratio for potassium.

In other words, the above decrease in heat transfer will probably be realized at about 1.13 T rather than at 1 T. Equation (26) is of course based on a very rough way of accounting for the fact that in reality the field will be uniform rather than spherical.

CONCLUSIONS

The paper consists of an order of magnitude phenomenological attempt to describe the magnetic field effect on nucleate boiling. Results have been obtained for the growth of the bubble and heat transfer rates. A number of approximations have been made the most limiting ones being the absence of the inertia forces and the one related to the geometry of the field. On the other hand, since the present findings are compared with the nonmagnetic ones, they probably provide a guide as to what trends to expect, in conducting an appropriate experiment. The work also delineates the non-dimensional parameters in terms of which the laboratory data can be presented with some pertinent reflection of the physics of the problem. A new non-dimensional parameter was discovered namely

$$K = \frac{\sigma_e B^2 \alpha}{\gamma \Delta T} = \frac{\sigma_e B^2 \alpha}{\Delta p}.$$

Physically this is the ratio of the ponderomotive force over the pressure forces that act on the bubble. On the other hand it is the parameter KJa^2 that determines the magnetic interaction. For a given fluid this parameter is proportional to the square of the magnetic field and the superheat ΔT.

The absence of inertia forces will be handled in a subsequent paper, but one can state an obvious conclusion. For very high magnetic fields the inertia forces produce negligible contributions even though in the non-magnetic cases the inertia forces might play a major role in the initial stage of the bubble growth. The presence of the inertia forces destroys the parabolic character of the growth and therefore it destroys also the possibility of defining an appropriate time independent Reynolds number. Of course, a "local in time" Reynolds number can be developed and hence a "local in time" Nusselt number can be computed. Eventually, as time passes and the bubble increases in size the phenomenon is "heat transfer controlled" and the inertia forces do not enter into the discussion.

Finally the assumption of a constant spherical magnetic field will need to be reviewed for a more detailed and accurate description of the bubble growth.

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REFERENCES

1. A. P. Frass, Analysis of a recirculating lithium blanket designed to give a low magnetohydrodynamic pumping power requirement, USAEC Report ONL-TM3756 (1972).
2. V. J. Lunardini, An experimental study of the effect of a horizontal magnetic field on the nucleate pool boiling of water and mercury with 0.02% magnesium and 0.0001% titanium, Ph.D. Thesis, Ohio State University (1963).
3. O. C. Faber and Y. Y. Hsu, The effect of a vertical magnetic induction on the nucleate boiling of mercury over a horizontal surface, *Chem. Engng Prog. Sym. Ser.* **64**, No. 82, 33–42 (1968).
4. A. P. Fraas, D. B. Lloyd and R. E. MacPherson, Effects of a strong magnetic field on boiling of potassium, ORNL-TM-4218 (1973).
5. H. Kawamura, M. Seki, Y. Shiina and K. Sanokawa, Experimental studies on heat transfer by natural convection and pool boiling of sodium in a strong magnetic field, *J. Nucl. Sci. Technol.* **12**, 280–286 (May 1975).
6. L. E. Scriven, On the dynamics of phase growth, *Chem. Engng Sci.* **10**(1/2), 1–18 (1959).
7. M. S. Plesset and S. A. Zwick, The growth of vapor bubbles in super heated liquids, *J. Appl. Phys.* **25**, 493 (1954).
8. T. G. Theofanous and P. Patel, Universal relations for bubble growth, *Int. J. Heat Mass Transfer* **19**(4), 425–429 (1976).
9. K. Forster and N. Zuber, Dynamics of vapor bubbles and boiling heat transfer, *A.I.Ch.E. Jl* **1**(4), 531–535 (1955).
10. L. Y. Wagner and P. S. Lykoudis, The effect of liquid inertia on bubble growth in the presence of a magnetic field, Paper No. AIChE-37, Proceedings of the ASME, AIChE 16th National Heat Transfer Conference, St. Louis, Missouri (8–11 August, 1976).

CROISSANCE DES BULLES EN PRESENCE D'UN CHAMP MAGNETIQUE

Résumé—Le problème de la croissance d'une bulle en présence d'un champ magnétique est examiné selon l'hypothèse d'un processus contrôlé par le transfert thermique. Les présents travaux font apparaitre l'existence d'un nouveau nombre sans dimension qui représente, du point de vue physique, le rapport des forces de pesanteur aux forces de pression calculé sur la base d'une échelle de longueur et d'un temps relatifs au coefficient de diffusion thermique. On trouve que dans le cas d'un champ magnétique sphérique, la croissance de la bulle est encore parabolique dans le temps mais le taux de croissance est plus faible. Des estimations du transfert thermique faites suivant la méthode de Forster et Zuber indiquent que le transfert de chaleur en ébullition nucléée se trouve réduit en présence de champs magnétiques. Dans un exemple numérique, on montre que cette réduction est plus importante pour le potassium que pour le mercure.

BLASENWACHSTUM IN ANWESENHEIT EINES MAGNETFELDES

Zusammenfassung—Das Problem des Blasenwachstums in Anwesenheit eines Magnetfeldes wird unter der Annahme untersucht, daß der Vorgang durch den Wärmeübergang bestimmt wird. Die Arbeit zeigt, das Vorhandensein einer neuen dimensionslosen Größe, welche physikalisch das Verhältnis der ponderomotorischen Kräfte zu den Druckkräften darstellt; dieses Verhältnis wird unter Verwendung des Temperaturleitkoeffizienten in Abhängigkeit von Ort und Zeit berechnet. Es zeigte sich, daß das

Blasenwachstum in Anwesenheit eines sphärischen Magnetfeldes weiterhin parabolisch über der Zeit verläuft, die Wachstumsrate jedoch geringer ist. Der Wärmeübergang beim Blasensieden wird durch die Anwesenheit magnetischer Felder vermindert, wie eine Abschätzung nach der Methode von Forster und Zuber ergab. Anhand eines numerischen Beispieles wird gezeigt, daß bei Kalium eine stärkere Verringerung eintritt als bei Quecksilber.

РОСТ ПУЗЫРЯ В ПРИСУТСТВИИ МАГНИТНОГО ПОЛЯ

Аннотация — Исследуется задача роста пузыря в присутствии магнитного поля при допущении, что определяющим процессом является перенос тепла. В ходе исследования вскрыто наличие нового безразмерного числа, которое физически описывает отношение пондеромоторных сил к силам давления, рассчитанных по шкале длины и времени, отнесенных к коэффициенту термодиффузии. Найдено, что для сферического магнитного поля рост пузыря во времени происходит по закону параболы, но скорость роста замедляется. Оценка теплообмена проводилась по Форстеру и Зуберу, согласно которым интенсивность теплообмена при пузырьковом кипении уменьшается при наличии магнитных полей. В численном отношении это уменьшение более существенно для калия, чем для ртути.